Computation with Absolutely No Space Overhead

Lane Hemaspaandra[1] Proshanto Mukherji[1] Till Tantau[2]

[1]Department of Computer Science
University of Rochester

[2]Fakultät für Elektrotechnik und Informatik
Technical University of Berlin

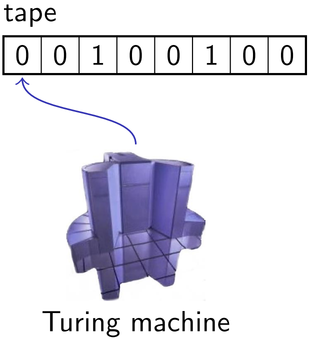
Developments in Language Theory Conference, 2003

The Model of Overhead-Free Computation
- The Standard Model of Linear Space
- Our Model of Absolutely No Space Overhead

The Power of Overhead-Free Computation
- Palindromes
- Linear Languages
- Context-Free Languages with a Forbidden Subword
- Languages Complete for Polynomial Space

Limitations of Overhead-Free Computation
- Linear Space is Strictly More Powerful

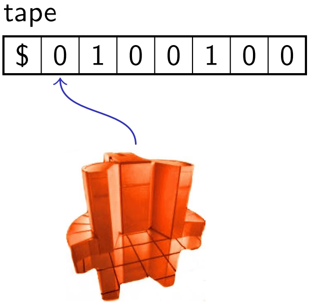
The Standard Model of Linear Space



Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

The Standard Model of Linear Space

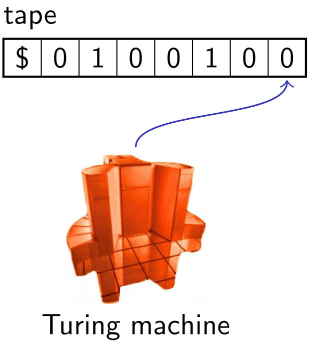


Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

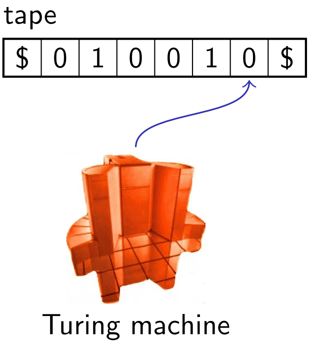
The Standard Model of Linear Space



Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

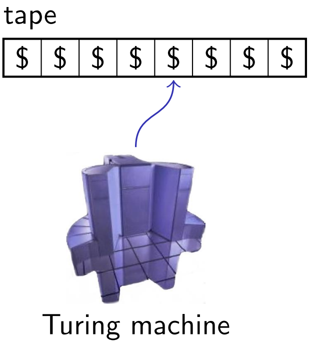
The Standard Model of Linear Space



Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

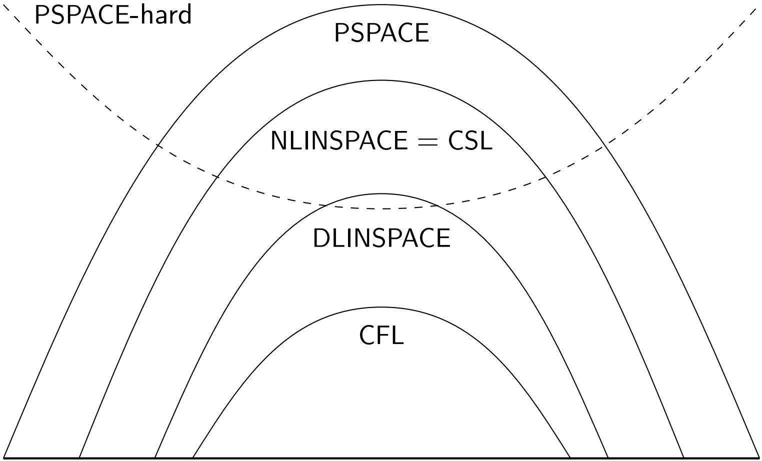
The Standard Model of Linear Space



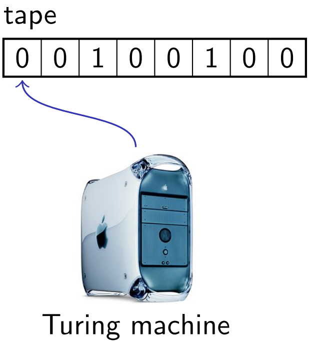
Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet is larger than input alphabet

Linear Space is a Powerful Model



Our Model of “Absolutely No Space Overhead”



tape

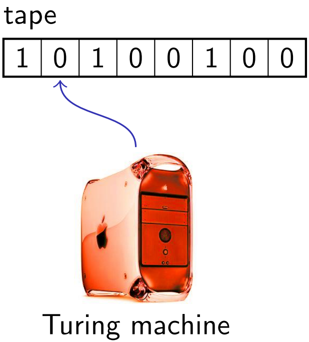
0	0	1	0	0	1	0	0

Turing machine

Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet equals input alphabet

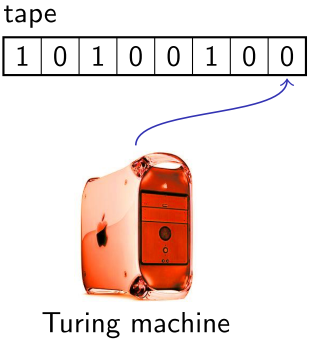
Our Model of “Absolutely No Space Overhead”



Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet equals input alphabet

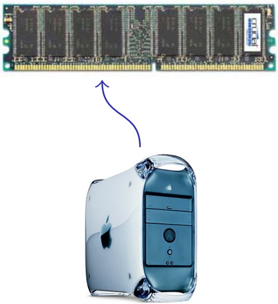
Our Model of “Absolutely No Space Overhead”



Characteristics

- Input fills fixed-size tape
- Input may be modified
- Tape alphabet equals input alphabet

Our Model of “Absolutely No Space Overhead”



Turing machine

Intuition

- Tape is used like a RAM module

Definition of Overhead-Free Computations

Definition

A Turing machine is overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet.

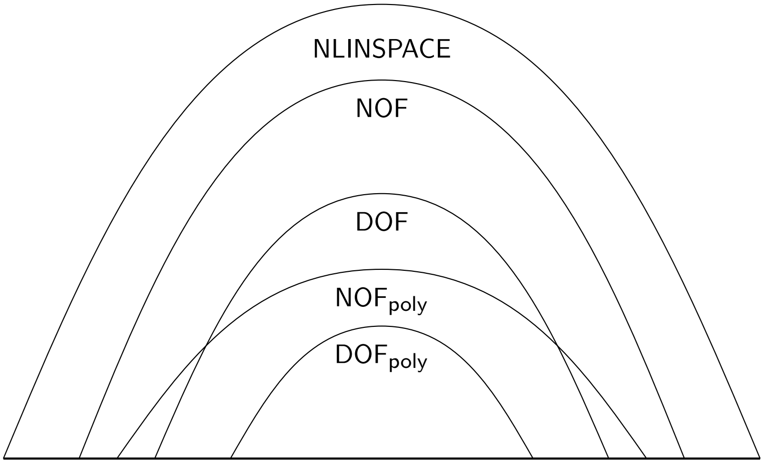
Overhead-Free Computation Complexity Classes

Definition

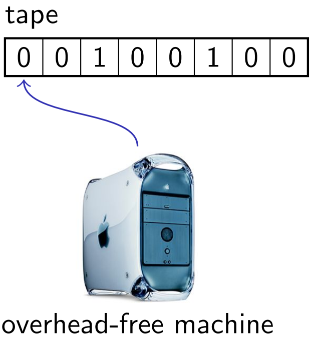
A language $L \subseteq \Sigma^*$ is in

- DOF if L is accepted by a deterministic overhead-free machine with input alphabet Σ,
- DOF_{poly} if L is accepted by a deterministic overhead-free machine with input alphabet Σ in polynomial time,
- NOF is the nondeterministic version of DOF,
- NOF_{poly} is the nondeterministic version of DOF_{poly}.

Simple Relationships among Overhead-Free Computation Classes



Palindromes Can be Accepted in an Overhead-Free Way

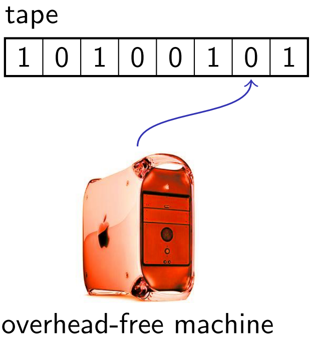


Algorithm

Phase 1:
Compare first and last bit
  Place left end marker
  Place right end marker

Phase 2:
Compare bits next to end markers
  Find left end marker
  Advance left end marker
  Find right end marker
  Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way



Algorithm

Phase 1:

Compare first and last bit

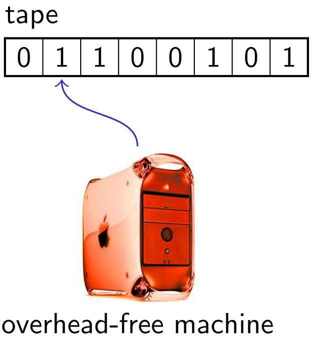
- Place left end marker
- Place right end marker

Phase 2:

Compare bits next to end markers

- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

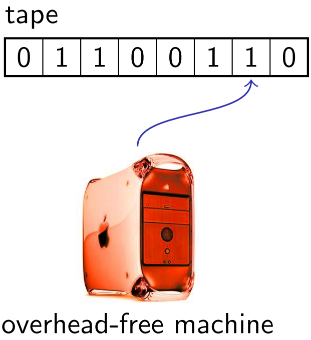


Algorithm

Phase 1:
Compare first and last bit
  Place left end marker
  Place right end marker

Phase 2:
Compare bits next to end markers
  Find left end marker
  Advance left end marker
  Find right end marker
  Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way



Algorithm

Phase 1:

Compare first and last bit

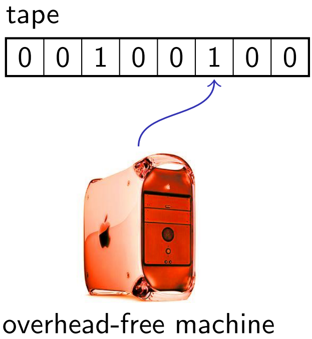
- Place left end marker
- Place right end marker

Phase 2:

Compare bits next to end markers

- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

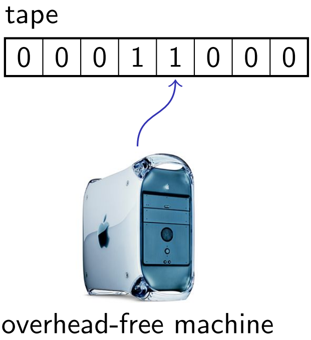


Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Palindromes Can be Accepted in an Overhead-Free Way

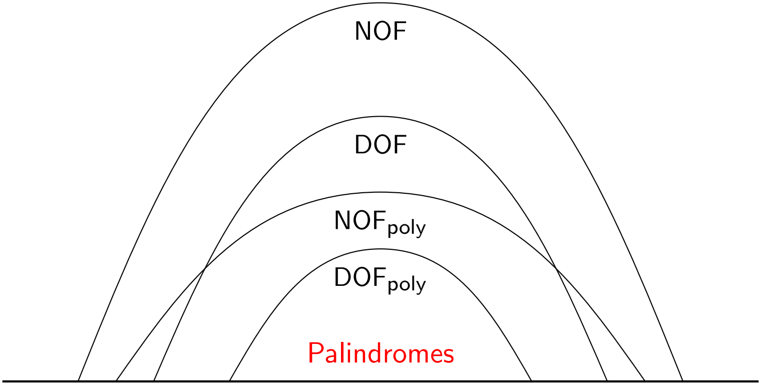


Algorithm

Phase 1:
Compare first and last bit
- Place left end marker
- Place right end marker

Phase 2:
Compare bits next to end markers
- Find left end marker
- Advance left end marker
- Find right end marker
- Advance right end marker

Relationships among Overhead-Free Computation Classes


NOF
DOF
$\mathsf{NOF}_{\mathsf{poly}}$
$\mathsf{DOF}_{\mathsf{poly}}$
Palindromes

A Review of Linear Grammars

Definition

A grammar is linear if it is context-free and
there is only one nonterminal per right-hand side.

Example

G_1: $S \longrightarrow 00S0 \mid 1$.
G_2: $S \longrightarrow 0S10 \mid 0$.

Definition

A grammar is deterministic if
"there is always only one rule that can be applied."

Example

G_1 is deterministic.
G_2 is not deterministic.

Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every deterministic linear language is in $\mathsf{DOF}_{\mathsf{poly}}$.

Continued Review of Linear Grammars

Definition

A language is metalinear if it is the concatenation of linear languages.

Example

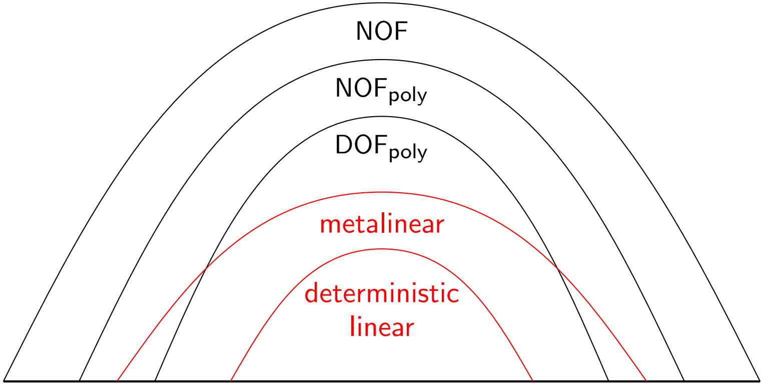
$\textsc{triple-palindrome} = \{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$.

Metalinear Languages Can Be Accepted in an Overhead-Free Way

Theorem

Every metalinear language is in $\mathsf{NOF}_{\mathsf{poly}}$.

Relationships among Overhead-Free Computation Classes


NOF
$\mathrm{NOF}_{\mathrm{poly}}$
$\mathrm{DOF}_{\mathrm{poly}}$
metalinear
deterministic linear

Definition of Almost-Overhead-Free Computations

Definition

A Turing machine is almost-overhead-free if

- it has only a single tape,
- writes only on input cells,
- writes only symbols drawn from the input alphabet plus one special symbol.

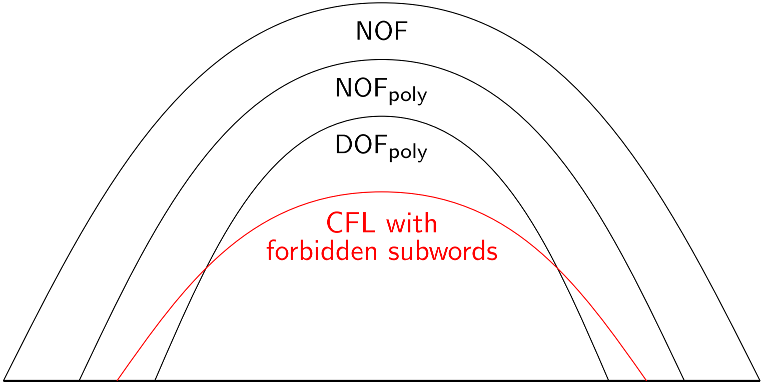
Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

Theorem

Let L be a context-free language with a forbidden word.
Then $L \in \mathrm{NOF}_{\mathrm{poly}}$.

The proof is based on the fact that every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.

Relationships among Overhead-Free Computation Classes


NOF
NOF$_{\text{poly}}$
DOF$_{\text{poly}}$
CFL with
forbidden subwords

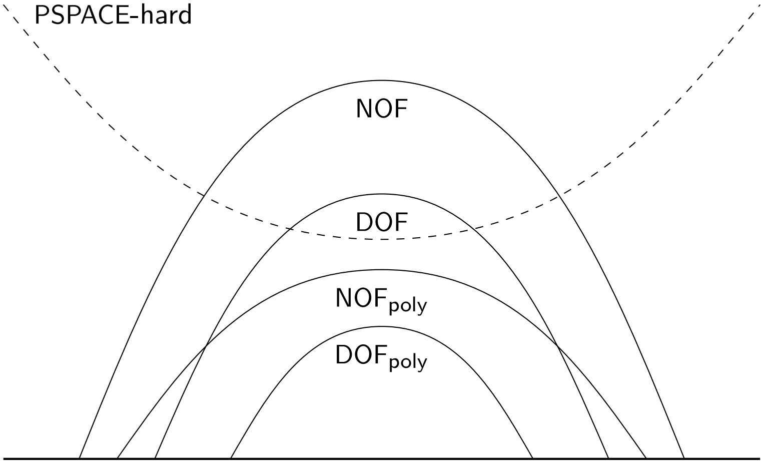
Some PSPACE-complete Languages Can Be Accepted in an Overhead-Free Way

Theorem

DOF contains languages that are complete for PSPACE.

The proof is based on the fact that for every $L \in \text{DLINSPACE}$ there exists an isometric homomorphism h such that $h(L) \in \text{DOF}$.

Relationships among Overhead-Free Computation Classes


PSPACE-hard
NOF
DOF
NOF$_{\text{poly}}$
DOF$_{\text{poly}}$

Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

Theorem

$\mathrm{DOF} \subsetneq \mathrm{DLINSPACE}$.

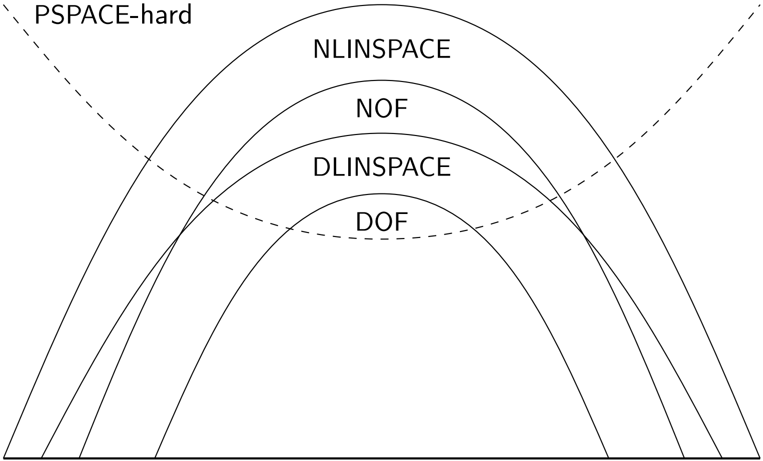
Theorem

$\mathrm{NOF} \subsetneq \mathrm{NLINSPACE}$.

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.

Relationships among Overhead-Free Computation Classes

Linear Space is Strictly More Powerful



Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

Conjecture

DOUBLE-PALINDROMES $\notin$ DOF.

Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin$ NOF.

Proving the first conjecture would show DOF $\subsetneq$ NOF.

Summary

- Overhead-free computation is a more faithful model of fixed-size memory.
- Overhead-free computation is less powerful than linear space.
- Many context-free languages can be accepted by overhead-free machines.
- We conjecture that all context-free languages are in $\mathrm{NOF}_{\mathrm{poly}}$.
- Our results can be seen as new results on the power of linear bounded automata with fixed alphabet size.